Propensities, Correlations and Metaphysics[1]

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1. Introduction

In his extensive writings[3] on the philosophy of quantum mechanics (QM) ~~Popper~~ Sir Karl Popper has always tried to understand the theory in terms of a metaphysical blueprint that combines realism with indeterminism. By realism I mean here the thesis that microphysical systems at all times possess sharp, definite values for all their physical attributes, which are simply discovered by appropriate measurement procedures. These attributes obtain quite independently of human observers and their conscious activities. Measurements may indeed change the values of a physical magnitude so the state ~~before~~ of a microsystem prepared by measurement may differ from the state before measurement, and such changes may come about in an irreducibly indeterministic way. As a result it may be impossible to prepare homogeneous ensembles of microsystems in which every member of the ensemble has identical values for all the associated physical attributes. This for Popper is the real significance of the uncertainty relations. They express objective scatter relations imposing limitations on the possibility of preparing homogeneous ensembles of microsystems.

new para [But since for Popper, measurements are just a particular type of physical interaction between a microsystem and ~~[illegible]~~ a macroscopic measuring apparatus, the inherent indeterminism associated with the time-evolution of physical magnitudes must be regarded as a pervasive feature of quantum ~~[illegible]~~ reality.

Schematically the picture one has of quantum reality on Popper's account is one in which at any particular time all physical attributes have sharp values but in general these values evolve in time in an ineluctably stochastic fashion. The probability of finding a particular value at a particular time for any particular physical magnitude is specified by the quantum-mechanical state — the state is just a catalogue of all those objective propensities for physical magnitudes to have specific

Obs. This metaphysical picture of quantum reality is clearly a very attractive one, but it is widely held to be flawed by commitment to the existence of ~~[illegible]~~ instantaneous action-at-a-distance between widely separated physical systems. Crudely speaking, such interaction is believed to be demonstrable from the existence of correlations between physical magnitudes associated with the two physical systems. This if true this would provide prima facie problems for ~~[illegible]~~ reconciling QM and special relativity, standardly understood as prohibiting causal interaction between space-like separated events.

new para [The question to be addressed in this paper is: What can one learn about causality from the existence of correlations

# Footnote ~~3~~ ~~4~~ 5. We are allowing positive and negative causes in our analysis, according as $\text{Prob}(\underline{a}/\underline{b}) > \text{Prob}(\underline{a}/\sim\underline{b})$ or $\text{Prob}(\underline{a}/\underline{b}) < \text{Prob}(\underline{a}/\sim\underline{b})$. Note also that $\text{Prob}(\underline{a}/\underline{b}) > \text{Prob}(\underline{a}/\sim\underline{b})$ is equivalent to $\text{Prob}(\underline{a}/\underline{b}) > \text{Prob}(\underline{a})$. Similarly $\text{Prob}(\underline{a}/\underline{b}) < \text{Prob}(\underline{a}/\sim\underline{b})$ is equivalent to $\text{Prob}(\underline{a}/\underline{b}) < \text{Prob}(\underline{a})$

⊙ Insertion where $\underline{b}$ is an event temporally earlier than $\underline{a}$

O Insertion ~~any~~ where ~~where~~ $\underline{b}$ is an event temporally earlier ~~earlier~~ than ~~b~~ $\underline{a}$

Insertion ★ For ~~$\underline{a}$~~ $\underline{b}$ temporally later than ~~$\underline{b}$~~ $\underline{a}$ this explication of cause does not apply due to Lewis's prohibition on backtracking counterfactuals.

in an irreducibly indeterministic or stochastic setting?

The elucidation of probabilistic causality between events has been widely discussed in the literature[(1),(3),(2),4].
The basic idea is that events a and b are putatively related by probabilistic causality iff Prob $(\underline{a}/\underline{b}) \neq$ Prob $(\underline{a}/\sim\underline{b})$, i.e. changing b to ~b changes the objective chance of event a ~~obtaining~~ happening.[5]

A word about notation: the argument of a probability function should really be understood as a proposition. We shall use symbols such as a and b to denote distinct events and also the propositions that express the happening of a, b etc. Sometimes we shall use a to denote a generic ~~result~~ event such as a physical magnitude ~~measurement~~ taking a value ~~result~~. The specific ~~result~~ event that ~~the~~ ~~that the~~ magnitude ~~generic generic measurement result~~ a has the value Ea will be denoted by '$\underline{a} = E_a$', ~~which in the appropriate context will also denote the proposition 'the result of measuring a is Ea'~~.

For the interpretation of conditional probability we shall generally follow Lewis(7). in understanding Prob$(\underline{a}/\underline{b}) = p$ as ~~short hand for~~ explicated by

Eberhard ⑥ (7)

$$\underline{b} \;\Box\!\!\rightarrow\; (\mathrm{Prob}(\underline{a}) = p) \qquad (1)$$

where $\Box\!\!\rightarrow$ signifies the Lewisian counterfactual conditional. * Eberhard

non-trivial

Probabilistic causality between a and b ~~would~~ i.e. b as a probabilistic cause of a would then be analysed as a combination of (1) and

Insertion
* We shall refer to (1) and (2) ourselves as *counterfactual relevance*, since we are going later to introduce an additional condition for causal dependence. But for the moment we continue to expound Lewis's ideas.

$$\sim \underline{b} \;\Box\!\!\rightarrow\; \sim(\text{Prob}(\underline{a}) = p) \qquad (2)$$

(1) and (2) actually characterize what Lewis refers to as causal dependence between $\underline{a}$ and $\underline{b}$ *Insertion. ~~In general~~ Lewis notes that causal dependence implies causation, but the converse is not in general true since causal dependence, unlike causation, is not necessarily a transitive relation. But Lewis connects the general notion of causation with that of causal dependence via ~~an~~ the idea of a causal chain of successive events such that between each adjacent pair the relation of causal dependence obtains.

new para [Finally it should be noted that (1) and (2) are a special case of causal dependence between families of events.

~~$\forall \varepsilon_b \left[(\underline{b} = \varepsilon_b) \;\Box\!\!\rightarrow\; \text{Prob}(\underline{a} = p_{\varepsilon_b}) \; (\text{Prob}(\underline{a}) = p_{\varepsilon_b}) \right]$~~

where ~~p_{ε_b} is the~~

$$\forall \varepsilon_a \, \forall \varepsilon_b \left[(\underline{b} = \varepsilon_b) \;\Box\!\!\rightarrow\; \left(\text{Prob}(\underline{a} = \varepsilon_a) = p^{\varepsilon_a}_{\varepsilon_b} \right) \right] \qquad (3)$$

where $p^{\varepsilon_a}_{\varepsilon_b}$ is the numerical conditional probability of obtaining the value ε_a for $\underline{a}$ given that $\underline{b}$ has the value ε_b, and we assume that $\forall \varepsilon_a$, $p^{\varepsilon_a}_{\varepsilon_b}$ has distinct numerical values for distinct values of ε_b. This condition is clearly satisfied in a Bell-type experiment where $\underline{a}$ and $\underline{b}$ refer to ~~denote~~ the ~~measurements~~ ~~of~~ spin-projections parallel to directions $\hat{a}$ and $\hat{b}$

for two widely-separated particles in the singlet state Ψ_S of their total spin, for all angles between $\hat{a}$ and $\hat{b}$ other than a right angle. Thus, in ~~For~~ this case it is easily verified that with two possible values ± 1 for each of E_a, E_b, we have.

$$\left.\begin{aligned} p^{+1}_{+1} &= \cancel{\sin\cos^2\tfrac{1}{2}\theta_{ab}}\ \sin^2\tfrac{1}{2}\theta_{ab} \\ p^{+1}_{-1} &= \cancel{\cos\sin^2\tfrac{1}{2}\theta_{ab}}\cdot\cos^2\tfrac{1}{2}\theta_{ab} \\ p^{-1}_{+1} &= \cancel{\cos\sin^2\tfrac{1}{2}\theta_{ab}}\ \cos^2\tfrac{1}{2}\theta_{ab} \\ p^{-1}_{-1} &= \cancel{\sin\cos^2\tfrac{1}{2}\theta_{ab}}\cdot\sin^2\tfrac{1}{2}\theta_{ab} \end{aligned}\right\} \qquad (4)$$

where θ_{ab} is the angle between the vectors $\hat{a}$ and $\hat{b}$.

So except in the case $\theta_{ab} = \pi/2$ we have *putative* 'stochastic causality' between the generic events $\underline{a}$ and $\underline{b}$.

new para [I say 'putative' at this stage because (3) may fail. the 'correlations might well result from a 'common cause' associated with the 'source' of the singlet state. But Bell's work showed that the common cause explanation of the correlations could never account for their precise mathematical form ~~of the correlations~~. We are left at this stage with two alternatives

(1) a common cause overlaid by a direct cause

Of course we must be careful that the variation in the way the cause is brought about does not include the initiation of other possible influences on the effect. If Jill strikes the match and pours a bucket of water over it then it does make difference to the occurrence of the explosion, as compared with Jack who just strikes the match. We shall return to this important reservation later. But for the moment

(2) Just a direct cause between a and b (Diagram 4)

In Ref. (10) ~~my (1986)~~ and subsequent publications I followed option (2) and raised the question of whether in such a scenario we should conclude that ~~in this case~~ a and b are causally connected. I argued for an additional necessary condition for causation, ~~viz.~~ a condition which I called 'robustness'.

2. The Robustness Condition ~~I shall~~

I shall begin by motivating the robustness condition in the context of classical physics. The idea in its simplest terms is that causes produce their effects independently of how the causes themselves are brought about. If striking a match causes an explosion then it does not matter if Jack strikes the match or Jill strikes the match. If striking is the cause, then Jack and Jill etc are simply irrelevant. Put a little more technically causes screen off their effects from the causal antecedents of the causes.

Let us take a simple example from classical physics to illustrate the main idea. ~~The In Kirchhoff's solution~~ In solving of the initial value problem for the homogeneous scalar wave equation the amplitude of the wave at any ~~spatial~~ space-time point is expressed as an ~~surface~~ integral over ~~a closed spatial surface surrounding~~ all space

~~* Footnote. Wave propagation in two dimensions is gno does not satisfy Huygen's principle – see Morse and Feshbach (1953) pp 842–843.~~

~~o [illegible] (which include normal gradients and time derivatives of the amplitude)~~

of the of the initial value at some earlier time amplitude and its time derivative 7

[illegible]

~~the point in question of field variables evaluated at the appropriate retarded times~~ multiplied by suitable Green's functions, describing in physicist's parlance, how the initial ~~surface~~ ~~distortion amplitudes~~ variables are propagated to produce the resultant amplitude at the space-time point in question. The initial ~~surface~~ values of the amplitude and its time-derivative ~~field~~ 'cause' ~~at~~ ~~at the retarded~~ all times earlier than ~~times~~ 'cause' the resultant amplitude. But we do not have to enquire how these initial ~~surface~~ values were produced. Given, but they are what they are the wave equation fills in the rest of the causal story for us in terms of the solution of the relevant initial ~~boundary~~ value problem.

New Para [Note that in a linear problem of this sort each element of field variable ~~surface~~ is regarded as an independent cause, the total effect being just the sum of the effects of ~~all~~ the independent causes. In a non-linear problem one might still want to talk of partial causes, but each partial cause produces an effect which depends on how the other partial causes are operating. If one draws a distinction here between cause and conditions one might want to characterize the situation in the non-linear case by saying that each partial cause plays the role of a condition in respect of how every other partial cause is operating.

6

(·) ~~Footnote ¥. Technically the equation describing the motion of the drumhead in this situation is an integro-differential equation as compared with the differential equation describing the motion of the drumhead in vacuo. — see Morse and Feshbach Ref. (13) 14 15 (1953) pp898-9. for details.~~

We remind the reader that

¥ footnote: [Wave propagation in two space dimensions does not satisfy Huygens' Principle. See Morse and Feshbach (1953) pp. 842-3.

For purposes of this discussion

II footnote [this curve ~~does not~~ is supposed not to cancel until the rim of the drumhead new para

Let us now take a slightly more complicated case that will illustrate the problems involved in a general analysis. Consider the vibration of a drumhead which is itself coupled to produce sound waves in the surrounding air. The amplitude at a given point on the drumhead at a given time can now be expressed as the sum of two contributions ~~effects~~. (1) ~~the amplitudes~~ (and derivatives) on a closed curve # surrounding the point in question propagating in accordance with the two-dimensional wave equation for the drumhead * . (2) the effect of sound waves originating from displacements of the drumhead impinging back on the drumhead. But (2) can only be calculated if we know the amplitude (and derivatives) at appropriate times everywhere on the drumhead. In particular we notice that in this problem, ~~i.e.~~ then the boundary values ~~amplitudes~~ involved in (1) are produced in turn by other causal processes (the sound waves) which contribute to the final effect. footnote ⊙

So it is clear that, in some cases, the 'causes of causes' are not irrelevant to the final effect, since they may themselves involve other causal ~~processes contributing to the final effect~~

~~* See footnote on back of p. 7.~~

~~⊙ See footnote 6 on back of p. 7.~~

⊙ Footnote 6. Technically the equation describing the motion of the drumhead in this situation is an integro-differential equation as compared with the differential equation describing the motion of the drumhead in vacuo — see Ref. 15 pp. 898-9 for details.

(1) the initial values of the amplitudes and its time derivative at an earlier time propagating forward in time in accordance with the two-dimensional wave equation for the drumhead *

(2) the effect of sound waves ~~arriving~~ originating from the changing displacements of the drumhead impinging back on the drumhead after this initial time.

~~But (2) can only be calculated~~

But ~~for initial times sufficiently close to the time at which we are trying to evaluate the amplitude~~ the contribution from (2) can only be calculated if we know the time derivative of the amplitude ~~of its time derivatives~~ at still earlier times, so in general we can say that the way in which the initial values in (1) are produced initiates other causal processes (the sound waves) which contribute to the final effect. 6

[illegible]

Thus it ~~So, it~~ is clear that, in some cases at least, the 'causes of causes' are not irrelevant to the final effect as we have ~~seen~~ just said ~~since they may themselves initiate other causal processes contributing to the final effect.~~

Let us try to generalize from these examples. ~~Following Healey (1991) we may introduce a notion of internal robustness (IR): A stochastic relation between two events a and b is internally robust just in case Prob (a / b) is invariant under all modifications in causal antecedents of b which leave b fixed and which ~~are~~ preserve causal antecedents of a which are neither causal antecedents nor causal consequents of b.~~

Quantor
$\forall$

Let us suppose firstly that all the events in
the actual world consist of physical magnitudes
taking values

~~do not mediate the their effect on $\underline{a}$ entirely via their effect on $\underline{b}$. The excluded causal antecedents of $\underline{a}$ which whose modification is not countenanced in the definition, are just those whose modification may change the probability of $\underline{a}$ even if $\underline{b}$ is held fixed, and hence provide apparent counterexamples to the preliminary formulation of robustness.~~

~~Our formulation of interval robustness differs significantly from that of Healey, in order to take account of criticisms of the original formulation advanced by Nancy Cartwright, that we must exclude modifications of joint common causes of $\underline{a}$ and $\underline{b}$.~~

Insertion ★

In the case of probabilistic causality we cannot expect Prob$(\underline{a}|\underline{b})$ to be robust if the way $\underline{b}$ is brought about can affect the probability of $\underline{a}$ occurring. It is only when $\underline{b}$ is the *total* cause of $\underline{a}$ that such a condition would be appropriate.

But suppose $\underline{b}$ can itself be broken down into a conjunction of *partial* causes $b_1, b_2, \ldots$ etc. then we would require Prob$(\underline{a} / \underline{b_1} \wedge \underline{b_2} \wedge \ldots)$ to be robust, i.e. independent of the way in which $\underline{b_1}, \underline{b_2} \ldots$ etc are brought about. What this means is that if $\underline{b_1}\ \underline{b_2}, \underline{b_3} \ldots$ are *fixed* at magnitudes $\varepsilon_{b_2}, \varepsilon_{b_3} \ldots$ then Prob$(\underline{a} = \varepsilon_a / (\underline{b_1} = \varepsilon_{b_1}) \wedge (\underline{b_2} = \varepsilon_{b_2}) \wedge (b_3 = \varepsilon_{b_3}) \ldots)$ is a robust function of the quantities $\varepsilon_a, \varepsilon_{b_1}, \varepsilon_{b_2}, \varepsilon_{b_3} \ldots$.

* ~~Situation so d is a common effect of two independent causes $\underline{Z_1}$ and $\underline{Z_2}$~~

* Situations which arise from variation in the numerical magnitude of the causal antecedents of some or all of the Z_i as they occur in the actual world. (~~this~~ the underlined qualification is essential to prevent problems of the Jill-and-the-pail-of-water variety referred to above)

i.e. the function is form-invariant under all ways of varying about these values ~~ε_{b_1} for b_1, b_2, b_3, etc which do not involve any alteration in the values for b_2 b_3~~ etc.

for

new para

[This is quite a different condition from claiming that $\forall b_i$, Prob $(\underline{a} = \varepsilon_a \mid \underline{b}_i = \varepsilon_{b_i})$ is robust.

To illustrate the distinction consider the following causal structure ~~, where the full lines joining events~~ ~~or~~

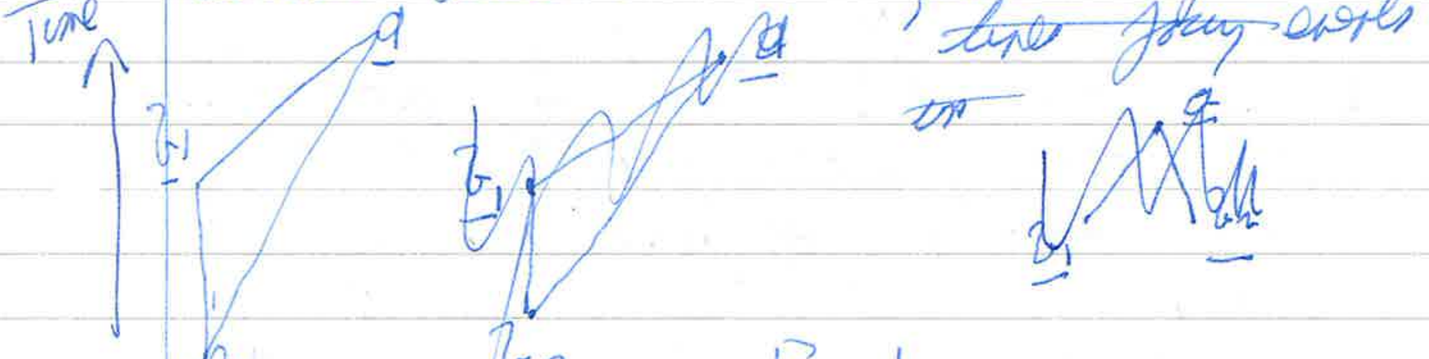


Fig. 1.

where the full lines joining events indicate causal connections ~~* [illegible]~~

~~Suppose to give a concrete case, b_2 deterministically causes b_1, so~~

~~Then Prob $(\underline{b}_1 = \varepsilon_{b_1} \mid \underline{b}_2 = \varepsilon_{b_2}) = 1$ for some specified ε_{b_2} and ε_{b_1}.~~

(for example)

Then, Prob $(a = \varepsilon_a \| b_1 = \varepsilon_{b_1})$

$$= \sum_{\varepsilon_{b_2}^{\beta}} \text{Prob}\left(a = \varepsilon_a \| (\underline{b}_1 = \varepsilon_{b_1}) \wedge (\underline{b}_2 = \varepsilon_{b_2}^{\beta})\right) \times \text{Prob}\left(b_2 = \varepsilon_{b_2}^{\beta} \mid \underline{b}_1 = \varepsilon_{b_1}\right) \qquad (5)$$

~~$= \sum_{\varepsilon_{b_2}^{\beta}}$ Prob $(a = \varepsilon_a \mid b_1 = \varepsilon_{b_1} \wedge b_2 = \varepsilon_{b_2}^{\beta}) \times$ Prob $(b_2 = \varepsilon_{b_2})$ $\times$ Prob $(b_1 = \varepsilon_{b_1} \mid b_2 = \varepsilon_{b_2}^{\beta}) \times$ [illegible] $\times$ Prob $(b_2 = \varepsilon_{b_2}^{\beta})$ / Prob $(b_1 = \varepsilon_{b_1})$~~

~~$=$ Prob $(a = \varepsilon_a \mid b_1 = \varepsilon_{b_1} \wedge b_2 = \varepsilon_{b_2})$ $\times$ Prob $(b_2 = \varepsilon_{b_2})$ / Prob $(b_1 = \varepsilon_{b_1})$ Prob $(b_1 = \varepsilon_{b_1})$~~

* Insertion

Suppose now we alter the way in which $\underline{b}_1 = \varepsilon_{b_1}$ and $\underline{b}_2 = \varepsilon_{b_2}$ are brought about, so ~~as~~ that Prob$(\underline{a} = \varepsilon_a / (\underline{b}_1 = \varepsilon_{b_1}) \cap (\underline{b}_2 = \varepsilon_{b_2}))$ is fixed (by our robustness condition) but also in such a way that [~~Prob $(\underline{b}_1 = \varepsilon_{b_1} / \underline{b}_2 = \varepsilon_{b_2})$ and Prob $(\underline{b}_2 = \varepsilon_{b_2})$ remain unchanged, but Prob $(\underline{b}_1 = \varepsilon_{b_1})$ is increased. Prob $(\underline{b}_1 = \varepsilon_{b_1})$ and Prob $(\underline{b}_2 = \varepsilon_{b_2})$ gets are changed. Then in general it is clear from (1) that in general Prob $(\underline{a} = \varepsilon_a / \underline{b}_1 = \varepsilon_{b_1})$ will change.~~] Prob$(\underline{b}_2 = \varepsilon_{b_2} / \underline{b}_1 = \varepsilon_{b_1})$ gets changed. Then it is clear from (8) that in general Prob$(\underline{a} = \varepsilon_a / \underline{b}_1 = \varepsilon_{b_1})$ will change.

Insertion

~~Suppose now we alter the way in which $b_1 = \epsilon_{b_1}$ is brought about, so as to keep Prob($a = \epsilon_a \mid b_1 = \epsilon_{b_1} \wedge b_2 = \epsilon_{b_2}$) and Prob($b_2 = \epsilon_{b_2}$) fixed but so as to increase Prob($b_1 = \epsilon_{b_1}$). In this case the above equation shows that Prob($b_1 = \epsilon_{b_1}$) Prob($a = \epsilon_a \mid b_1 = \epsilon_{b_1}$) will decrease.~~

new para

Our generalization of the robustness condition to composite causation is related to the idea of internal robustness introduced by Healey(16) ~~in his (1990)~~. Healey's requirement reads as follows: ~~The~~ A stochastic relation between two events a and b is internally robust just in case Prob($a \mid b$) is invariant under all modifications ~~of~~ in causal antecedents of b which leave b fixed and which preserve causal antecedents of a which are neither causal antecedents nor causal consequents of b.

new para

But Healey's definition cannot deal with the situation illustrated in Fig. 1 where b_2 is a common cause of b_1 and of a. This criticism of Healey's condition on partial causes has been particularly stressed by Cartwright(17) ~~(1991a)~~. Our own condition is designed to deal with all ~~the~~ those cases covered adequately by Healey together with those cases where Cartwright's criticisms ~~are cogent~~ apply.

~~contd on p.13~~

~~* Insertion Finally we must check for robustness.~~

~~II Insertion This relevance is robust~~

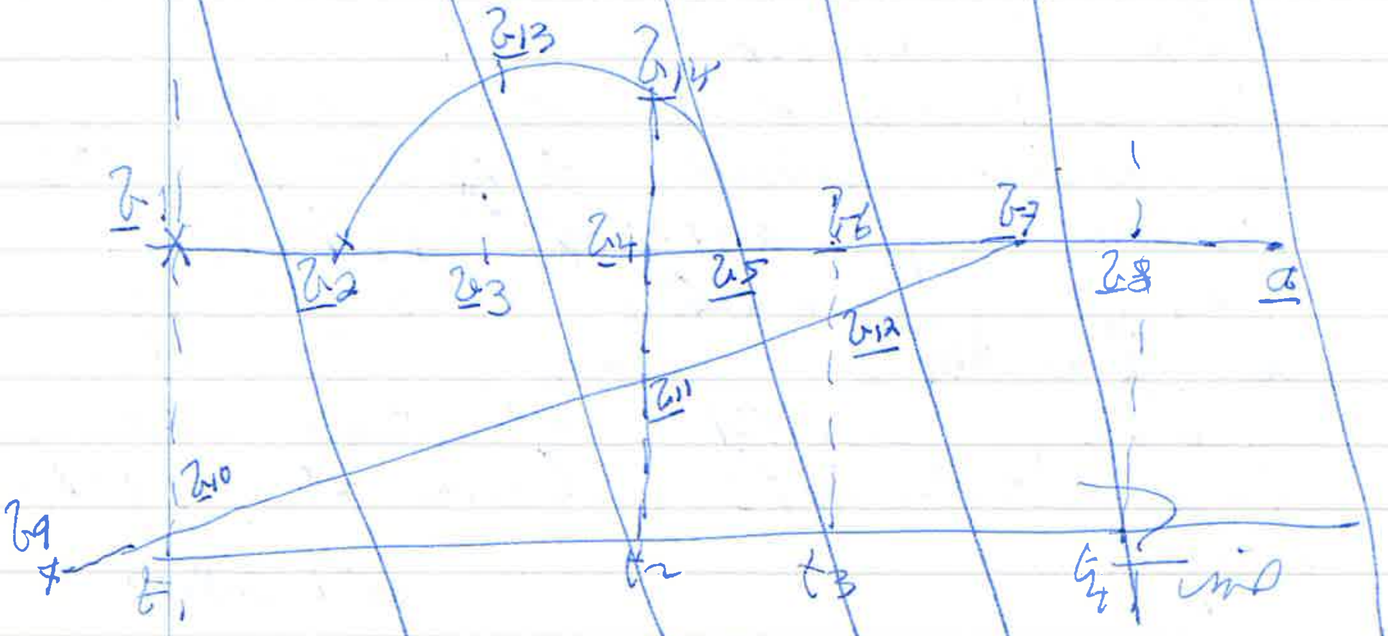
~~⊙ Insertion See Insertion (A)~~

⊡ Insertion (or any combination of them)

(Twice)

from p.13

~~* Insertion Finally if b_i is [any one of] the temporally latest of the partial [positive] causes, we must check that the set $\{b_1, b_2 ---\}$ cannot be augmented by any additional events [a simple [illegible] with] prior to b_i without violating the first two conditions. (This is essentially the robustness condition). Clearly the set of partial causes is by no means unique [illegible]. We illustrate some of the complexities in a full analysis by considering the following simplified structure causal structure leading up to the event a.~~



~~Contd~~ Fig. 2

~~Cont. Insertion (B)~~

new para. [There are some additional points we must attend to. Firstly how do we identify partial causes of 'd' such as b_1, b_2 ..etc.? We just have to apply the ~~statistical~~ counterfactual relevance criterion (3) to b_1, keeping b_2, b_3 - - fixed, then to b_2 keeping b_1, b_3 - - fixed' and so on *! ~~But we must also~~ This means ~~show~~ that none of b_2, b_3 .. etc [1] greater screen off d from b_1, that none of b_1, b_3 .. etc screen off d from b_2 and so on.

greater [1]

~~* Further (Central) [illegible]~~

~~Informally speaking b_1 is a partial cause of d just in case b_1 is statistically relevant to d [illegible] but this it relevance is not simply mediated through other putative partial causes.~~ ~~[illegible] ①~~

~~But does this mean that the robustness condition is analytically true since any counterexample can be absorbed by appropriate expansion of the relevant partial causes collection of partial causes? The substantive claim of~~

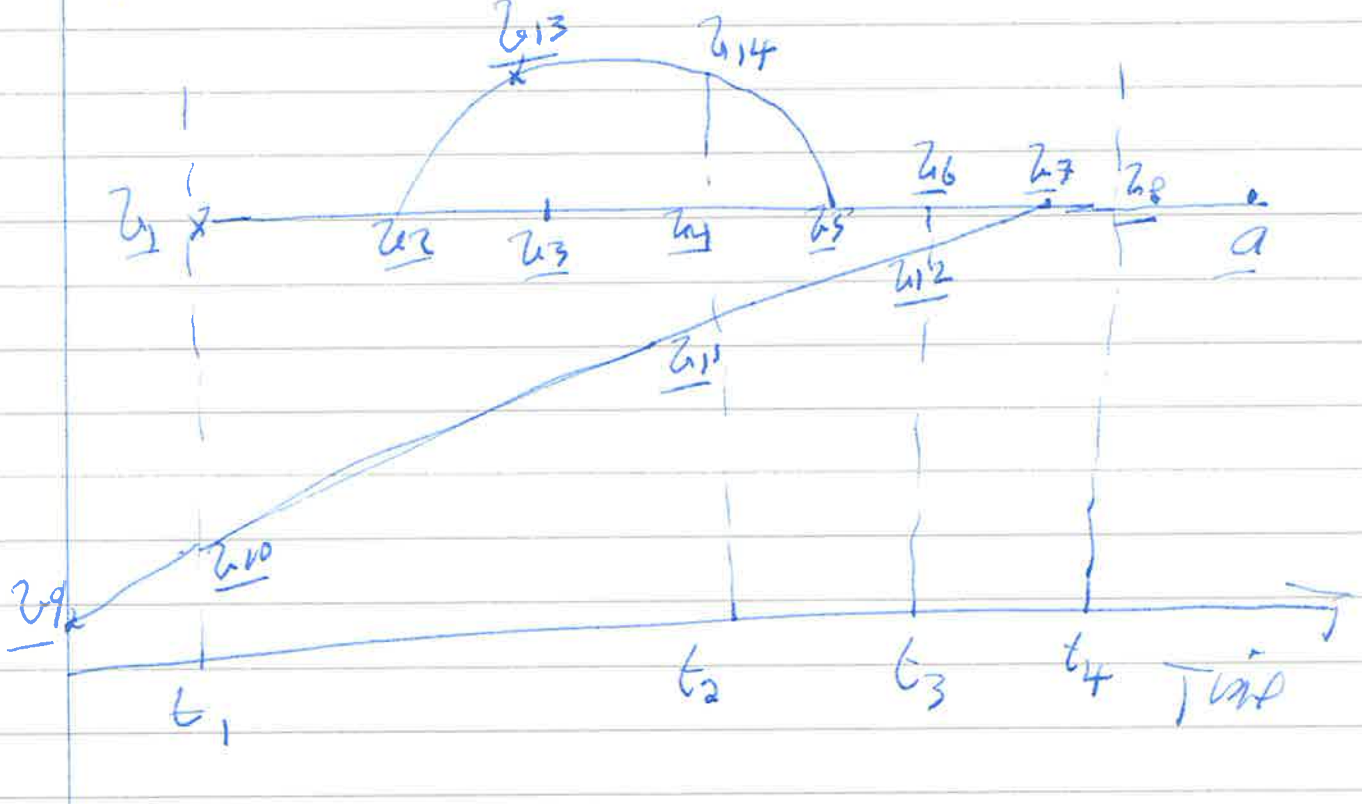
~~the robustness condition is really an existential one that there exists a determinate limit to the escalating nesses of partial causes that there remains a non-empty set of alternative histories leading up to the total cause under which the probability of the effect conditionalized on the total cause is robust (i.e. invariant).~~

~~If this existential condition were violated we would live in what I characterized in my (198) as a 'marshmallow' world, in which total causes are total histories. This is, I concede, a possible world, but~~

Footnote 7 The connection between partial causes and Mackie's INUS analysis of causation(18) should be clear.

Secondly, ~~Finally~~ if b_i is any one of the temporally latest of the partial causes, we must check that the set $\{b_1, b_2, \ldots\}$ cannot be augmented by any additional events prior to or simultaneous with b_i without violating the first ~~two~~ conditions. (This is ~~essentially~~ the robustness condition.).

Clearly the ~~minimal~~ set of partial causes is by no means unique[†]. We illustrate some of the complexities in a full analysis by considering the following simplified causal structure leading up to the event a.



b_{13}
b_{14}
b_1
b_2
b_3
b_4
b_5
b_6
b_7
b_8
a
b_{12}
b_{11}
b_{10}
b_9
t_1
t_2
t_3
t_4
Time

Fig. 2.

~~Insertion (A)~~ In Fig 2 three Markov chains are ~~shown~~ illustrated, ~~comprising~~ with ~~ever~~ selected events shown. The first chain includes e_1, e_2, e_3, e_4, e_5, e_6, e_7, e_8 and q. The second chain ~~includes~~ branches off from the first at e_2, includes e_{13} and e_{14} and re-unites with the first chain at e_5. The third chain includes e_9, e_{10}, e_{11}, e_{12} and merges with the first chain at e_7.

To construct maximal sets of partial causes of q start with e_1 at time t_1. This can be augmented according to our rules, with e_9 so the set $\{e_1, e_9\}$ is a possible maximal set of partial causes of q. But we could include e_{10}, simultaneous with e_1, which screens off e_9 which can then be dropped, so $\{e_1, e_{10}\}$ is another maximal set of partial causes. Now suppose we start with e_4. This can be extended to a maximal set by including e_{11} and e_1, but e_1 could equally be substituted by e_2, e_{13} or e_{14}. Notice here a general point in this kind of example. At any given time there exists a maximal set of partial causes all happening at that time and corresponding to the relevant events on all the causal chains operating at that time. Notice that we cannot supplement the set $\{e_1, e_4, e_{11}\}$ with e_3 due to the prohibition on backtracking in assessing the counterfactual relevance of e_2 to q via the sequence e_3, e_2, e_{13} e_{14} e_5 e_6 e_7 e_8.

Ref. ~~[illegible]~~ 19.

Footnote 8/6 cf the discussion in ~~Chase & Cox~~ p 267 ff. where it is shown how non-Markovian processes arise because of incomplete specification of the instantaneous state of a Markov process.

~~Footnote 7. for Tool of p. 12. The connection between partial causes and Mackie's (18) INUS analysis of causation should be clear.~~

Moving forward to t_3, z_6 and z_{12} provide a ~~maximal~~ possible minimal set, while at t_4, z_8 alone is the cause of $\underline{q}$.

new para

In this example, which is typical of the situation in classical physics, it is clear that while 'causes of causes' may be relevant to producing the final event $\underline{a}$ the prior causes always leave instantaneous traces at the later time ~~[illegible]~~ so ~~some~~ it is somewhat misleading to say that the history of a cause must ~~can~~ be necessarily involved in accounting for the total effect. This is relevant to oft-cited examples such as hysteresis in the phenomenon of ferromagnetic magnetization. The susceptibility ~~can be~~ depends on the instantaneous domain structure in the sample, which thus screens off the effect of the prior magnetic history.

new para

But suppose robustness is violated. Can it always be restored by appropriate expansion of the relevant collection of partial causes?

Copied Contd p. 24

footnote

* ~~Note that we are allowing positive, negative~~
~~or that~~

~~one in which the notion of cause would be pragmatically useless since we clearly cannot manipulate the total history leading up to an event.~~

~~It is I believe a substantive claim about our 'world' that it is not of the 'marsh-mallow' variety. Robustness ~~then~~ condition is not so much a necessary condition on causation as an ontological conception but rather a contingently necessary feature of our own world.~~

I go on to argue: ~~The answer to this is~~ 'No'. From a God's eye point of view all the possible partial causes are 'laid out' so to speak. ~~[there are then two possibilities: (1) robustness for all the partial causes is satisfied or (2) robustness fails for at least one of them. If (2) is the case there are two possible reactions: (A) we have not included all the partial causes, (B) the partial causes come in two varieties those satisfying robustness and those that do not. Now (A) is ruled out by our metaphysical hypothesis. With regard to (B)]~~ If we find an event that looks like a partial cause, but fails the robustness test, then 'under our metaphysical assumption' there are no additional partial causes that can be brought in to rescue robustness. So in such a situation we must conclude that although the ~~[illegible]~~ counterfactual relevance of the event makes it look like an instance of causation it is in fact a new sui generis metaphysical category.

This does not mean that from our epistemological point of view it may not be a useful methodological imperative when faced with an apparent failure of robustness to look for further partial causes.

But as we shall argue in the quantum-mechanical case of the EPR correlations that methodological imperative may actually lead us ~~lead us~~ into other sorts of metaphysical problem, in particular how to reconcile the space-like character of the interval between the causally related events and the constraints of special relativity.

In this situation a possible line to take is to resist the methodological imperative which works perfectly well in the arena of classical physics, and embrace the idea of non-robust correlations which in virtue of their very non-robustness, are non-causal in character.'

new para

{It should be noted that even in the classical physics examples, robustness always is restored at some level, so there exists a determinate limit to the escalating nexus of partial causes, and moreover there remains a non-empty set of alternative histories leading up to the total cause even when the probability of the effect conditionalized on the total cause is robust (i.e. invariant).

* footnote 9 see footnote 11.

II Smetton provided us also support liberal with the notion of event.

Note

* [illegible] wrote in
"Explanation" in D. Knowles (ed.)
Explanation and its Limits (Cambridge University
Press, Cambridge, 1990) p.148

If this existential claim were violated we would live in what I characterized in * ~~Ref. (11)~~ Ref. 14 ~~Ng (198~~) as a marshmallow world in which total causes ~~are~~ total histories. No doubt this is a metaphysical possibility, but it is not what is at issue in the debate about EPR correlations. In the EPR case it is easy to restore robustness as we shall see in a moment #. The question is, do we need to ~~want to~~ or want to?

new para [Next I want to turn to the question of 'small disturbances'. When I originally introduced the idea of robustness (~~#~~10) I included a clause stating that robustness should obtain under sufficiently small disturbances of the way in which the cause was brought about. The intuitive idea here was that large disturbances might well invoke other partial causes contributing to the effect. But the issue of partial causes can now be dealt with in the way we have described above. So with this more comprehensive analysis there is no need to introduce a 'small disturbances' clause in the definition of robustness. We shall exploit this ~~relaxation of the~~ ~~weakening~~ strengthening of the condition in a much simplified discussion of the QM case below #9

new para [Finally ~~I want~~ in this section, I want to say something about the 'mark method for identifying causal processes.

Ref. 10.

~~Ref (19)~~

In ~~my (19—)~~ I claimed Reichenbach's mark method(20) depended on the notion of robustness. What I had in mind was the following. If a causal process mediates action between event *b* and event *d*, then it is possible to change the way in which *b* is brought about by marking it with another event *m* in such a way that *d* still occurs {with the same probability} (robustness) but the mark also survives (carried by the causal process).

Of course some marks may rub off, in which case they are simply inappropriate to the mark criterion, or the mark ~~mark~~ may also interact causally with the production of the effect. In such a case the test is not identifying the original causal process and other marks should be tried: ~~Jones~~ Cartwright and Jones(21) ~~(19—)~~, cite the case of ~~[illegible]~~ ringing birds which may affect their chances of achieving a migration. But to give a test that unringed birds causally achieve migration with a certain definite probability we need to ring them more lightly so the ringing does not affect their flight!

* Example of two widely separated particles emerging from a common source.

II Footnote 10 For a more comprehensive discussion of Butterfield ~~(19~~ ~~II 18(19)~~ ~~(20)~~ ~~(21)~~ (22).

3 QM Correlations and Robustness

~~We~~ revert now to the case of Bell-type experiments involving correlations between spin-projections ~~of widely separated particles~~ along suitably ~~[illegible]~~ chosen directions* insertion and in an appropriately selected QM state Ψ_s for the spin variables of the particles.

On the realist view we are pursuing the most obvious explanation of these correlations would lie in a common cause associated with the previous specification of the hidden-variable state of the source of the particles. Denoting the events comprising the spin-projections for the two particles taking values by $\underline{a}$ and $\underline{b}$ and the hidden-variable state of the source taking a value by $\underline{\lambda}$ then we are assuming a screening off condition, $\mathrm{Prob}^{\Psi}(\underline{a}/\underline{b} \wedge \underline{\lambda}) = \mathrm{Prob}^{\Psi}(\underline{a}/\underline{\lambda})$. But this assumption (with others we do not need to discuss for our present purposes) leads to the Bell inequality for the correlations between $\underline{a}$ and $\underline{b}$ for appropriate different choices of $\underline{a}$ and $\underline{b}$ which is famously violated by experiment! So this looks as though $\underline{b}$ and $\underline{\lambda}$ are partial causes of $\underline{a}$. But since the dependence of $\mathrm{Prob}(\underline{a}/\underline{b} \wedge \underline{\lambda})$ on the values of its arguments is a robust function of ~~$\underline{b}$ and~~ these ~~arguments~~ values the preferred causal model of the correlation between $\underline{a}$ and $\underline{b}$ seemed to ~~look like~~ as illustrated in Fig. 3. 10

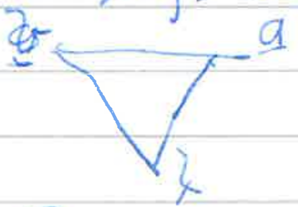


Fig. 3

footnote p. 11

* The proof of non-robustness for the singlet state of two spin-$\frac{1}{2}$ particles can be much simplified ~~(20)~~ ~~(22)~~ (23) by considering the Mermin ~~(18)~~ (~~22~~ (23) (24) (19)) version of the Greenberger – Horne – Zeilinger (GHZ) state, for three spin-$\frac{1}{2}$ particles ~~(19)~~, which is an eigenstate of $T = \sigma_x^1 \sigma_y^2 \sigma_y^3$ where σ_x^1 is the x-component of Pauli spin for the first particle, σ_y^2 the y-component of Pauli spin for the second particle, and so on, with eigenvalue +1 say.

Now perturb particle 2, so that its spin precesses through 180° around the z-axis, thus flipping σ_y^2 to $-\sigma_y^2$, in the Heisenberg picture. So $T \to -T$ with eigenvalue −1. Thus, in the Schrödinger picture, the effect of the perturbation is to change the ~~eigen~~ state from $|T=+1\rangle$ to $|T=-1\rangle$. But this means non-robustness since in the state $|T=+1\rangle$ $\mathrm{Prob}((\sigma_x^2 = 1)|(\sigma_y^2 = 1) \cap (\sigma_y^3 = 1)) = +1$, while in the state $|T=-1\rangle$ this conditional probability changes to zero, and so on. This simplified proof is due to Clifton(25)

But a direct causal link between a and b would pose relativistic problems as we mentioned in the introduction. So let us try a model in which there is only direct interaction, as illustrated in Fig. 14

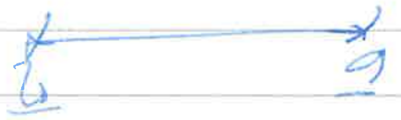


Fig. 14

~~Surely this would be no improvement.~~

We are modelling the conditional probabilities $Prob^{\psi}(a/b)$ as given in the QM state ψ by a direct interaction between a and b. But it is easy to demonstrate(10) that $Prob^{\psi}(a/b)$ is not a robust function of (its values) its arguments a and b under suitable perturbations of either open variable Fig 11 [illegible]. So does this show that the direct link between a and b is not causal and hence opens the possibility of peaceful coexistence with special relativity? Instead of causal order between b and a, we have a harmony or passion (as Shimony has called it(25)(26)).

New para [But perhaps more has been strongly resisted (10)(21) ~~(24 43)~~. After all what is going on in the proof of non-robustness is simply to demonstrate that ψ changes in such a way as to generate the change in , and hence non-robustness of the 'conditional probability'. What is being proposed is that ψ and b, for example, are to be regarded as two 'partial causes' of $\underline{a}$. Effectively we would replace Prob$^{\psi}(\underline{a}/\underline{b})$ by Prob$(\underline{a}/\underline{b} \wedge \psi)$ and then the action between $\underline{a}$ and $\underline{b}$ is again seen to be causal since robustness has been restored by the standard procedure of augmenting the set of partial causes.

new para [Accepting this argument depends crucially on how we interpret ψ and whether ψ corresponding to a certain ray in Hilbert space, is to be counted as an event. ~~('Insofar as ψ is an entangled state for the two separated particles one could argue that it is not a precisely localized property and its taking values' is hence not an event. But I would not myself want to tie the notion of event so closely to questions of localization)~~ In hidden-variable theories of the 'Bohm-type'㉔ ψ in its role as a 'quantum potential' seems sufficiently 'physical' to warrant its taking 'values' as events. But we are not concerned with the question – Can we avoid instantaneous causal action under all interpretations of ψ? Rather our claim is an existential one there exists some interpretation of ψ under which its taking 'values' could not reasonably be understood as an event. ~~This is~~ Such an interpretation is provided by ~~the~~ what I will call the *catalogue interpretation* of Ψ, already referred to briefly in the introduction. Ψ is to be regarded just as a catalogue of all the conditional probabilities Prob$(\underline{a} \mid \underline{b})$ for all values of $\underline{a}$ and $\underline{b}$. So on page one we list all the entries for Ψ_1, on page two all the entries for Ψ_2 and so on (We are imagining for purposes of exposition that the ~~number of~~ different QM states are denumerable, ~~as also the values for $\underline{a}$ and $\underline{b}$~~ and the possible values for a and $\underline{b}$ are finite in number!). But how can a catalogue be something over and above its entries, so that the change in the entries is 'explained' as 'caused' by the changes in the catalogue? Or even more absurdly, what sort of ~~event~~ is a physical event is a catalogue?! This interpretation of Ψ as a catalogue of probabilities or propensities as Popper would prefer to express it, is a perfectly reasonable and natural one, and happens also I would argue to avoid interpreting Bell-type correlations as involving space-like causality albeit of a probabilistic nature.

new para

I ~~now~~ now want ~~finally~~ to make some remarks on the connection between robustness and signalling. If we consider the simple case of a single cause $\underline{b}$ and its effect $\underline{a}$, then the marginal probabilities for $\underline{a}$ to take the value E_{a_i} and $\underline{b}$ to take the value ε_b, are related by

$$\text{Prob}(\underline{a}=\varepsilon_a) = \sum_{\varepsilon_b} \text{Prob}(\underline{a}=\varepsilon_a \mid \underline{b}=\varepsilon_b) \times \text{Prob}(\underline{b}=\varepsilon_b) \quad \text{— (6)}$$

Clearly if $\text{Prob}(\underline{a}=\varepsilon_a \mid \underline{b}=\varepsilon_b)$ is robust and we alter the way in which $\underline{b}$ takes the value ε_b in such a manner that $\text{Prob}(\underline{b}=\varepsilon_b)$ changes, then, in general, we can write

~~$\Delta\text{Prob}(\underline{a}=\varepsilon_a) = \sum_{\varepsilon_b} \text{Prob}(\underline{a}=\varepsilon_a \mid \underline{b}=\varepsilon_b) \times \Delta\text{Prob}(\underline{b}=\varepsilon_b)$ — (3)~~

~~relating changes in the marginal probabilities for $\underline{a}$ and $\underline{b}$.~~

~~unequal to zero~~

~~For given $\Delta\text{Prob}(\underline{b}=\varepsilon_b)$, it does not follow from (3) that $\Delta\text{Prob}(\underline{a}=\varepsilon_a)$ is unequal to zero. What is true is if the dependence of $\text{Prob}(\underline{b}=\varepsilon_b)$ is sufficiently under our control for all ε_b, then we can always choose $\Delta\text{Prob}(\underline{b}=\varepsilon_b)$ so that $\Delta\text{Prob}(\underline{a}=\varepsilon_a)$ gets changed and this allows the possibility of signalling~~

~~$\Delta\text{Prob}(\underline{a}=\varepsilon_a) = \left(\text{Prob}(\underline{a}=\varepsilon_a \mid \underline{b}=\varepsilon_b) - \text{Prob}(a=\varepsilon_a \mid \underline{b}\neq\varepsilon_b)\right) \times \Delta\text{Prob}(\underline{b}=\varepsilon_b)$~~

~~relating changes in marginal probabilities for a ε_a and b: ε_b.~~

$$\Delta\text{Prob}(a=\varepsilon_a) = \sum_{\varepsilon_b} \text{Prob}(\underline{a}=\varepsilon_a) \mid \underline{b}=\varepsilon_b) \times \Delta\text{Prob}(\underline{b}=\varepsilon_b) \quad \text{— (7)}$$

relating changes in the marginal probabilities for $\underline{a}$ and $\underline{b}$.

For given $\Delta\text{Prob}(\underline{b}=\varepsilon_b)$ it does not in general follow to

* Substitute, X012

Remember for given ε_a, all of Prob$(\underline{a}=\varepsilon_a | \underline{b}=\varepsilon_2)$ are different. ~~Order~~ ~~For all but the last of these probabilities there'll Δ Prob$(\underline{b}=\varepsilon_2) > 0$ — so for the last $\Delta(\underline{b}=\varepsilon_2) < 0$.~~ Denote these probabilities by $c_1, c_2 \ldots c_n$, ordered so that $c_1 > c_2 \ldots > c_{n-1} > c_n$.

C Denoting the associated values of Δ Prob$(\underline{b}=\varepsilon_2)$ by $x_1 \ldots x_n$ where $x_1+x_2+\ldots x_n = 0$ choose $x_1 \ldots x_{n-1} > 0$, so $x_n = -(x_1+\ldots x_{n-1})$

Then Δ Prob$(\underline{a}=\varepsilon_a) = \sum_{j=1}^{n} c_j x_j > c_{n-1}(x_1+\ldots x_{n-1}) - c_n x_n$

$$= (x_1+\ldots x_{n-1})(c_{n-1}-c_n) > 0.$$

The degenerate case of this theorem with $n=2$ was proved in ~~ms (19~~) Ref. (13) 12.

from ~~(3)~~ 47 that ΔProb$(\underline{a} = \mathcal{E}_a)$ is assigned to zero. What is true is that if the ~~dependence~~ of variation of Prob$(\underline{b} = \mathcal{E}_b)$ is sufficiently under our control, then we can always choose ΔProb$(\underline{b} = \mathcal{E}_b)$ so that ΔProb$(\underline{a} = \mathcal{E}_a)$ gets changed and this allows the possibility of 'signalling'. ~~footnote~~ 12

But no-signalling is a ~~much~~ logically stronger condition than non-robustness, it implies a very special sort of non-robustness. So the no-signalling theorems of QM[13] are a less sensitive test of absence of causality than the non-robustness test. If we could signal but via a non-robust link we could still rule out a causal connection. But as ~~stated~~ remarked in ~~ref (17)~~ Ref. 13, to show that no events are connected by causality in this situation we would need to check that robustness cannot be restored by ~~a suitable redefinition of event~~ considering less precisely specified events, ~~such~~ for example that Prob$\left((\underline{a} = \mathcal{E}_{a1}) \vee (\underline{a} = \mathcal{E}_{a2}) \cancel{\vee (\underline{a} = \mathcal{E}_{a3})\ \text{etc.}} \,\middle|\, \{\underline{b} = \mathcal{E}_{b1} \vee \underline{b} = \mathcal{E}_{b2}\}\right)$ ~~is robust~~ is not robust, and so on.

Finally I would stress that the robustness test can be applied in situations where the marginal probabilities are not altered at all. This is exemplified in my original treatment of the perturbation of spin-½ states by c-number fields in the singlet state for the total spin of two spin-½ particles as also in the GHZ example referred to in footnote ~~9~~ 11.

Acknowledgements I am grateful to Nancy Cartwright, Martin Jones, Andrew Elby, Harvey Brown, Richard Healey, David Papineau, Rob Clifton, ~~Martin Jones~~ and Jeremy Butterfield for helpful discussion of the issues addressed in this paper, although none of the views argued for should be imputed to them.

Footnotes:

1. This paper is dedicated to Sir Karl Popper on the occasion of his ninetieth birthday.

2. Department of History and Philosophy of Science, University of Cambridge, Free School Lane, Cambridge CB2 3RH, ~~England~~ United Kingdom.

~~4.~~ 5. ~~3.~~ ~~4.~~ ~~5.~~ See end of p. 2.
6 ~~4~~ See end of p. ~~8~~ 9.
7. ~~5~~ " " ~~p. A1~~ p. 13.
~~8~~ ~~6~~ " " ~~p. A1~~ p. 132.
9. ~~7~~ see footnote ~~2~~ 11.
10. ~~8~~ see end of p 17.
11. ~~9~~ see end of p. 18.
12. ~~10~~ See end of p. 22.

3. ~~1~~. see for example Refs. ~~(1) and (2)~~ 1. and 2.

4. ~~2~~. See for example. Suppes(3), Skyrms(4), Salmon(5 ~~4~~), Cartwright(6 ~~5~~)(7 ~~6~~), Eells(8).

13. See for example Redhead(11 ~~10~~) p. 116.

[illegible] [illegible], Preface and Section 4.4.

* [illegible] (University of Notre Dame Press, Notre Dame, Indiana, 1989), p.145.

Chase and Cox.

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14. ⇒ See p.26.

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Ch. 4

Figure Captions

Fig 1. Partial Causes: $\underline{b}_1$ and $\underline{b}_2$ are partial causes of $\underline{a}$, $\underline{b}_3$ is the cause of $\underline{b}_1$.

Fig 2. Markov Chains and Partial Causes: three chains $b_1\, \underline{b}_2\, b_3\, b_4\, \underline{b}_5\, \underline{b}_6\, \underline{b}_7\, \underline{b}_8$, $\underline{b}_2\, \underline{b}_{13}\, \underline{b}_{14}\, \underline{b}_{15}$ and $\underline{b}_9\, b_{10}\, \underline{b}_{11}\, \underline{b}_{12}\, \underline{b}_7$ contribute to the event $\underline{a}$.

Fig 3. Causal triangle for spin-projections a and b and hidden state λ of the source in a Bell-type correlation experiment.

Fig 4. Direct interaction between spin-projections a and b in a Bell-type correlation experiment.